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# Low-Energy Scattering of Black Holes and $p$-branes in String Theory

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We discuss the low-energy dynamics of generalized extremal higher membrane black hole solutions of string theory and higher membrane theories following Manton's prescription for multi-soliton solutions. A flat metric is found for those solutions which possess $\kappa$-symmetry on the worldvolume.

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## 1. Introduction

The construction of soliton and black hole solutions of string theory and their connections with various dualities in string theory have been the subject of much recent activity (see [1] and references therein). The soliton solutions typically arise as extremal limits of two-parameter charged black hole solutions. These extremal black holes saturate a Bogomol'nyi bound between their ADM mass and charge, thus ensuring their stability, in analogy with the extremal Reissner-Nordstrom black holes of Einstein-Maxwell theory. In both the string and Einstein-Maxwell cases, the saturation of this bound is associated with the existence of a "zero-force" condition, which allows for the existence of multiple extreme black holes in a static configuration. In the string context, the saturation of the Bogomol'nyi bound is also associated with the existence of spacetime supersymmetry. A further generalization inherent in the string scenario is that these black hole solutions may have higher membrane structure, arising in a higher-dimensional spacetime.

Most of the solutions discussed in the string context are associated with an underlying "worldvolume" supersymmetry of a higher-membrane theory (generalizing worldsheet supersymmetry in string theory), called $\kappa$-symmetry, which ensures that the correct number of degrees of freedom arise in the theory. It turns out that these $\kappa$-symmetric solutions are also associated with the preservation the maximal amount of spacetime supersymmetry in various embeddings.

In this work, we start with the generalized extremal black holes of Shiraishi [2], and note that these include both $\kappa$-symmetric and non-$\kappa$-symmetric solutions. We then generalize these solutions further to arbitrary higher-membrane black holes embedded in arbitrary higher dimensions and discuss their connections with the various dualities in string and higher membrane theories.

In section 3 we follow Manton's method [3] for the computation of the low-velocity scattering of both the $\kappa$-symmetric and non-$\kappa$-symmetric generalized extreme higher-membrane black holes and find that this scattering is trivial (*i.e.,* zero dynamical force) precisely for the $\kappa$-symmetric solutions.

Finally, we discuss our results in section 4 as well as the intriguing possibility that some of the extremal black holes we discuss might correspond to elementary states in string theory.

## 2. Generalized extreme black hole solutions

Consider the action

$$I_4(1) = \int d^4x\sqrt{-g}\left(R - \frac{1}{2}(\partial\phi)^2 - \frac{1}{4}e^{-a\phi}F_2^2\right), \tag{2.1}$$

where $g_{MN}$ is the metric, $\phi$ the scalar dilaton and $F_2 = dA_1$ the Maxwell field strength. $a$ is an arbitrary constant. The generalized multi-extreme black hole solution is given by [2]

$$\begin{aligned} ds^2 &= -F(x)^{-2/(1+a^2)}dt^2 + F(x)^{2/(1+a^2)}d\vec{x}\cdot d\vec{x}, \\ \phi &= -\frac{2a}{1+a^2}\ln F, \\ [A_1]_0 &= \pm\sqrt{\frac{4}{1+a^2}}F, \\ F(x) &= 1 + \sum_{i=1}^{N}\frac{k_i}{|\vec{x}-\vec{x}_i|}, \end{aligned} \tag{2.2}$$

where $\vec{x}_i$ are the locations of the $N$ black holes. The mass $m_i = k_i/(1+a^2)$ and electric charge $e_i = 2k_i/\sqrt{1+a^2}$ of each black hole saturate the Bogomol'nyi bound $m_i^2 \geq e_i^2/4(1+a^2)$, thus ensuring the stability of the multi-black hole configuration.

The single black hole solution represents the extremal limit of a two parameter family of charged black hole solutions in which this bound is not saturated. One can equally well find magnetically charged solutions, where the magnetic charge is related to the electric charge via the Dirac quantization condition $eg = 2\pi n$, where $n$ is an integer. For the specific value of $a = \sqrt{3}$, the action couples to a sigma-model of a theory with a fermionic worldline symmetry called $\kappa$-symmetry, which ensures that the nonphysical degrees of freedom in the worldline theory are projected out (see [1] and references therein).

It is possible to generalize the above solutions in two ways. In the first step, we can consider point-like black holes in arbitrary higher-dimensional spacetime $D$. The four-dimensional action (2.1) is then replaced by a $D$-dimensional action via $\int d^4x \to \int d^Dx$ and $I_4(1) \to I_D(1)$. Note that the argument 1 in the action represents the dimension of the worldline swept out by the point-like black holes. In this case, the $\kappa$-symmetric solutions correspond to $a^2 = 2(D-1)/(D-2)$.

Furthermore[2], we can generalize the point-like black holes themselves to higher-membrane objects, with, say, $d-1$ spatial directions (called a $(d-1)$-brane) by coupling to an antisymmetric tensor $A_d$ with $d$ indices. Here the replacement takes the form

[2] The procedure is only outlined below. For details see [4].

$(1/4)F_2^2 \to (1/2(d!))F_{d+1}^2$, where $F_{d+1} = dA_d$ is the field strength associated with $A_d$ in further generalizing the action via $I_D(1) \to I_D(d)$.

A useful dimension to define is the "dual" dimension $\tilde{d} \equiv D-d-2$. Then for a $(d-1)$-brane in $D$ dimensions with "electric" charge $e_d$ arising from the antisymmetric tensor, the natural solitonic object is a $(\tilde{d}-1)$-brane carrying topological "magnetic" charge $g_{\tilde{d}}$ [1]. These again are related by a (higher-dimensional) Dirac quantization condition. In this most general case, the $\kappa$-symmetric solutions correspond to $a^2 = 4 - 2d\tilde{d}/(D-2)$.

The existence of these generalized dual solitons is in fact the basis for the various dualities in string and higher-membrane theories. An especially interesting example of a duality which is of current interest is string/string duality in $D = 6$ ($d = \tilde{d} = 2$). An interesting consequence of this duality on reduction to four dimensions is the interchange of two previously studied dualities: the target-space $T$ duality, which generalizes the compactification scale size duality in string theory, and the strong/weak coupling $S$ duality [5]. An important implication of the latter lies in the application of perturbative techniques in the strong coupling region of string theory, which is of special interest in the attempt to understand string theory as a theory of quantum gravity. Furthermore, the existence of these dualities is likely to point to a reformulation of string theory in which these dualities are manifest.

## 3. Metric on moduli space

In the absence of exact solutions for time-dependent multi-soliton or multi-extreme black hole solutions, Manton's method for the computation of the metric on moduli space for two or more solitons represents a good low-velocity approximation to the exact dynamics.

Manton's prescription for the study of soliton scattering may be summarized as follows: One begins with a static multi-soliton solution, and gives the moduli characterizing this configuration a time-dependence. One then finds $O(v)$ corrections to the fields by solving the constraint equations of the system with time-dependent moduli. The resultant time-dependent field configuration only satisfies the full time-dependent field equations to lowest order in the velocities, but provides an initial data point for the fields and their time derivatives. Another way of saying this is that the initial motion is tangent to the set of exact static solutions. An effective action describing the motion of the solitons is determined by replacing the solution to the constraints into the field theory action. The kinetic action so obtained defines a metric on the moduli space of static solutions, and the geodesic motion on this metric determines the dynamics of the solitons [3].

A calculation of the metric on moduli space for the scattering of extreme Reissner-Nordstrom black holes and a description of its geodesics was worked out by Ferrell and Eardley [6]. Shiraishi [7] then extended this work to black hole solutions of (2.1) in four dimensions and of $I_D(1)$ in $D$ dimensions, all for arbitrary $a$.

Following Ferrell and Eardley's technique, it is possible to compute this metric for the generalized, higher-membrane extreme black holes of the previous section [4]. It turns out that precisely for the $\kappa$-symmetric solutions, this metric is flat, and the kinetic Lagrangian describes noninteracting extreme black holes.

For the more general case, the direct computation of the metric is also possible, but rather tedious. A tremendous simplification occurs when one realizes that the low-velocity dynamics of an arbitrary $(d-1)$-brane extremal black hole embedded in $D$ dimensions are simply related to that of the dimensionally reduced point-like black hole in $D-d+1$ dimensions [4], essentially due to the fact that in both cases the dual dimension is the same and given by, $\tilde{d} = D - d - 2$. This can be seen schematically via

$$(D, d) \rightarrow (D-1, d-1) \rightarrow \ldots \rightarrow (D-d+1, 1), \tag{3.1}$$

where the first number in braces indicates the spacetime dimension and the second the the dimension of the worldvolume swept out by the higher-membrane black hole, and where the arrows indicate dimensional reduction of both dimensions. Notice that the last pair of dimensions represents a point-like black hole.

As the dynamics of the point-like solutions have already been worked out by Shiraishi [7], we obtain our answer quite easily, taking care to account for the proper volume terms for the higher-membrane black holes. Note that, in particular, the $\kappa$-symmetric solutions can be expanded or reduced only to other $\kappa$-symmetric solutions, since only these solutions scatter trivially.

As an example of nontrivial scattering for non-$\kappa$-symmetric extreme black holes, consider the case of two point-like black holes in $D = 4$ with $a = 1$ in (2.1). Then the low-velocity Lagrangian is given following Manton's method by [7]

$$L = -M + \frac{1}{2} M V^2 + \frac{1}{2} \mu v^2 \left(1 + \frac{2M}{r}\right), \tag{3.2}$$

where $M = m_1 + m_2$ is the total mass, $\vec{V} = \vec{v}_1 + \vec{v}_2$ is the center-of-mass velocity, $\mu = m_1 m_2 / M$ is the reduced mass, $\vec{v} = \vec{v}_2 - \vec{v}_1$ is the relative velocity and $\vec{r} = \vec{x}_2 - \vec{x}_1$ is the relative separation of the two black holes. This interaction in fact yields Rutherford scattering.

## 4. Discussion

The flat metric and consequent trivial dynamics for the $\kappa$-symmetric solutions is a somewhat surprising result, and is probably connected with the existence of flat directions in the superpotentials associated with the underlying $\kappa$-symmetric theories. Another possibility is that these solutions also possess the maximal amount of spacetime supersymmetry in certain embeddings, and this may also constrain the dynamics considerably. For example, if we embed the four-dimensional black holes in $N = 8$, $D = 4$ supergravity, only the $\kappa$-symmetric $a = \sqrt{3}$ black hole preserves four of the spacetime supersymmetries.

Another interesting idea connected with the dynamics of these solutions is the possibility that the point-like extremal black holes might correspond to massive elementary states in the string spectrum. Particles with mass greater than the Planck mass possess a Schwarzschild radius greater than their Compton wavelength, and so presumably may display an event horizon even in a quantum-mechanical framework. In the absence of a theory of quantum gravity, however, this notion is hard to test. Since string theory is a possible theory of quantum gravity and predicts the existence of massive states at the Planck scale, it provides a framework in which to test this conjecture. Duff and Rahmfeld [8] argued that certain massive string states in fact correspond to extremal black hole solutions of (2.1). More recently, dynamical evidence for this conjecture was found by the present authors [9](see also [10]), where the dynamics of the black holes according to the Manton method and as discussed in this paper were compared with the string-theoretic scattering amplitudes of the corresponding string states. Remarkably, these two seemingly disparate methods yield the same long-range scattering. These results then promise to shed light on the nature of string theory as a theory of quantum gravity.

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